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Effectiveness–number of transfer units relationships for heat exchanger complex flow arrangements

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Abstract—Heat exchanger performance (or terminal temperatures), size and fluid flow rates are related in a dimensionless form in terms of the exchanger effectiveness, number of transfer units and heat capacity rate ratio. Such relationships are essential for design and analysis of any heat exchangers. Over the last 100 years, many heat exchanger flow arrangements have been analyzed and reported in the literature. However, since 1969, several very powerful methods have been developed to analyze complicated flow arrangements for two-fluid recuperators. These are matrix formalism, chain rule, and rules for exchangers with one fluid mixed, among others. These methods are briefly summarized in the paper with illustrative examples. Using these methods, 18 new recuperator flow arrangements have been analyzed and the results are presented in closed-form formulas assuming constant overall heat transfer coefficient and fluid properties. The results summarized here together with those published in the open literature should then provide the reader with an idea as to where to concentrate future research efforts on the subject.

INTRODUCTION

HEAT EXCHANGER basic analysis is done either by the effectiveness–number of transfer units (ε–NTU) method, the log-mean temperature difference (LMTD) method, P–NTU method, or one of the variants of these methods [1–4].

The first reported investigation of the exchanger analysis for a condenser ($C_{min}/C_{max} = 0$) was outlined by Thompson (Lord Kelvin) in 1859 in a letter to Joule [5]. Mollier [6] analyzed single-pass counterflow and parallelflow exchangers. Nusselt [7] analyzed a single-pass crossflow exchanger with unmixed–unmixed fluids. Nagle [8] was the first investigator to analyze shell-and-tube exchangers with single and two-pass shells with multiple tube passes. Since then, a large number of exchanger flow arrangements have been analyzed and reanalyzed by various methods in about 100 papers. Some of the most important milestones are an in-depth analysis of multipass shell-and-tube and crossflow exchangers by Bowman *et al.* [9], the formal introduction of the ε–NTU method for the exchanger analysis by London and Seban [10], the decomposition of complex flow arrangements by simpler constitutive elements by a matrix algebra treatment by Domingos [11], and a reappraisal of the mean temperature difference by Gardner and Taborek [12]. Some powerful methods and concepts that can be used for the analysis of very complex exchanger configurations for which no closed-form solutions were available before were introduced in a series of papers [13–18]. These are the extension of the Domingos matrix formalism, the concept of flow reversibility, a relation between the effectiveness of overall parallelflow and counterflow multipass flow arrangements, and the chain rule method.

Recently Chen and Hsieh [19] published a method which they refer to as simple, general and systematic to determine the effectiveness and fluid temperatures for assemblies of heat exchangers. Their method can analyze series coupled 1–N, 2–N and miscellaneous assemblies by solving linear equations presumably numerically. Those 1–N and 2–N assemblies can be analyzed in closed form using the Domingos method [11]. Their method is rather limited because the constituent parts of the assembly are subject to the following restrictions:

- They are limited to exchangers with two inlet and two outlet streams. Thus, no J- or H-shell exchangers are allowed as components.
- The inlet and outlet streams of the constituents must be perfectly mixed. Therefore, the scheme cannot be used for multi-row air coolers with unmixed air between rows.
- All the constituents must have the same heat capacity rate ratio.
- Nodes at which a stream splits into two or more

NOMENCLATURE

C flow stream heat capacity rate, Wc_p [W °C^{-1}]
C^* heat capacity rate ratio, C_{min}/C_{max} [dimensionless]
c_p specific heat of fluid at constant pressure [J kg^{-1} °C^{-1}]
M thermal matrix relating outlet to inlet temperatures, such as equation (3) [dimensionless]
M_{ij} matrix element relating the outlet stream i to the inlet stream j, it is the coefficient of t_j in the expression for t'_i [dimensionless]
NTU total number of transfer units, $NTU = UA/C_{min}$, $NTU_1 = UA/C_1$, $NTU_2 = UA/C_2$ [dimensionless]
NTU_{1J} number of transfer units for the J exchanger, $(UA/C_1)_j$ [dimensionless]
n number of tube rows
P temperature effectiveness of the fluid stream, $P_1 = (t'_1 - t_1)/(t_2 - t_1)$, $P_2 = (t_2 - t'_2)/(t_2 - t_1)$, $P_1 = P_2 R_2$, $P_2 = P_1 R_1$ [dimensionless]
$p_1(x)$ effectiveness density of Fluid 1 when the inlet stream is unmixed and all other streams are mixed [dimensionless]
$p'_1(x')$ effectiveness density of Fluid 1 when the outlet stream is unmixed and all other streams are mixed [dimensionless]
R heat capacity rate ratio, $R_1 = C_1/C_2$, $R_2 = C_2/C_1$, $R_{1J} = R_1$ for the J exchanger [dimensionless]
t fluid inlet temperature [°C]
t' fluid outlet temperature [°C]
U overall heat transfer coefficient [W m^{-2} °C^{-1}]
W fluid mass flow rate [kg s^{-1}]
x normalized coordinate in the transverse direction of an unmixed inlet Fluid 1 stream (see Fig. 5(b)) [dimensionless]
x' normalized coordinate in the transverse direction of an unmixed outlet Fluid 1 stream (see Fig. 5(a)) [dimensionless].

Greek symbol
ε heat exchanger effectiveness [dimensionless].

Subscripts
$A, B, \ldots$ exchangers $A, B, \ldots$
c counterflow
max maximum
min minimum
p parallelflow
1,2 Fluids 1 and 2, respectively.

Superscript
$'$ outlet of the exchanger.

streams (or conversely, two or more streams merge into a single stream) are not allowed. Thus, for example, the assemblies required to build a J-, G- or H-shell exchanger out of pure parallelflow or counterflow exchangers, cannot be handled.

The chain rule method presented in ref. [18], and in this paper, overcomes all of the above restrictions of Chen and Hsieh [19].

In this paper, 18 new exchanger configurations are analyzed using the Domingos matrix formalism rules, Pignotti chain rule, and a relation between the effectiveness of overall parallelflow and counterflow multipass exchangers. The results are presented in terms of P_1, NTU_1 and R_1 groups rather than ε, NTU and C^* groups of Kays and London [20]. The reason for this choice is that there are two ε–NTU expressions for a stream unsymmetric two-fluid exchanger depending upon whether Fluid 1 is the C_{min} or C_{max} fluid [2]; see, for example, the case of a single-pass crossflow exchanger with one fluid mixed, the other unmixed [2]. We will need only one expression in terms of the P_1–NTU_1 relationship in which case R_1 varies from 0 to ∞.

In all of the results derived in this paper, the following usual idealizations for heat exchanger analysis are invoked:

- Steady-state operation.
- Zero heat losses to the surroundings.
- Fluid properties and heat transfer coefficients constant and uniform.
- Zero longitudinal wall conduction.
- If the inlet fluid stream is split into two or more streams before entering the exchanger, their temperature is the same; and if one of the fluids has two outlet streams, these streams are assumed to mix for the purpose of evaluating the overall effectiveness of the exchanger.
- For the TEMA H shell-and-tube exchanger, it is idealized that there is a large number of baffles, no flow bypassing or flow leakage exists, and the shell fluid is mixed in the direction transverse to the fluid flow in each compartment formed between the longitudinal baffle and the shell.

In the following, the methods of matrix formalism and chain rules are briefly described together with illustrative examples and the limitations. The P_1–NTU_1 formulas for new exchangers are summarized in Table 5.

MATRIX FORMALISM

The analysis of an assembly of heat exchangers was first conducted by Domingos [11] for exchangers with two inlet and two outlet mixed streams, and extended in ref. [13] for multiple inlet and outlet fluid streams and in ref. [16] for unmixed inlet and outlet fluid streams. The results are briefly summarized next.

The Domingos method

Domingos [11] introduced a method to couple two or more individual exchangers to determine the effectiveness for the assembly of exchangers. In this method, matrix transformation rules are applied using individual heat exchanger effectivenesses as building blocks. Domingos first introduced the concepts of the thermal matrix (he referred to it as the static transfer matrix) and the thermal transfer factor of a heat exchanger. Since the thermal matrix is sufficient for the heat exchanger analysis, we will introduce it only here.

Thermal matrix. Two outlet fluid temperatures can be presented in terms of two inlet fluid temperatures, using the definitions of exchanger temperature effectivenesses P_1 and P_2 as

$$t'_1 = (1-P_1)t_1 + P_1 t_2 \tag{1}$$

$$t'_2 = P_2 t_1 + (1-P_2)t_2. \tag{2}$$

These equations were represented in the matrix notation by Domingos [11] as

$$\begin{bmatrix} t'_1 \\ t'_2 \end{bmatrix} = \begin{bmatrix} 1-P_1 & P_1 \\ P_2 & 1-P_2 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = M \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}. \tag{3}$$

Using the relationship $P_2 = P_1 R_1$, the above 2×2 matrix can be represented as

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} = \begin{bmatrix} 1-P_1 & P_1 \\ P_1 R_1 & 1-P_1 R_1 \end{bmatrix}. \tag{4}$$

Thus, if the two inlet temperatures and the matrix M are known, the two outlet temperatures can be obtained from equation (3). Hence, M is designated here as the basic thermal matrix. Note that the sum of the matrix elements in each row is equal to unity. The individual matrix elements have two subscripts i and j. The first subscript i labels the outlet stream, and the second one the inlet stream. For example, $M_{12} = P_1$ is the coefficient of t_2 in the expression for t'_1 as also can be found from equation (1). Similarly $M_{21} = P_1 R_1$ is the coefficient of Fluid 1 inlet temperature (t_1) in the expression for Fluid 2 outlet temperature (t'_2). The other matrix elements can also be interpreted in the same way. This interpretation is summarized in Table 1. We will make ample use of Table 1 while applying the chain rule to be discussed later for the exchanger analysis.

Table 1. Matrix elements for the four possible combinations of inlet and outlet streams

	Fluid 1 inlet	Fluid 2 inlet
Fluid 1 outlet	$1-P_1$	P_1
Fluid 2 outlet	$P_1 R_1$	$1-P_1 R_1$

It should be emphasized that as long as the idealizations mentioned in the Introduction are valid, the elements of the thermal matrix M are functions of P_1 and R_1 (and hence also of NTU_1), and they are *not* dependent upon the magnitude of inlet temperatures.

Also, if the fluid inlet temperature distribution is *not* uniform, the fluid outlet temperatures may not be related to the inlet temperatures by the simple relationships of equations (1) and (2). For example, see the matrix elements for the last case in Table 2 presented later. In such a case, the elements of matrix M may not contain P_1 or P_2 explicitly, but the relation between the outlet and inlet temperatures, which involves an integration over a continuous variable x, can still be put in matrix form (see equation (23) later).

Heat exchangers with series and parallel coupling

Domingos [11] used the thermal matrix formulation for individual component exchangers and arrived at the overall effectiveness of n exchangers of non-identical size (NTU_is) when connected in series or parallel coupling.

For n series coupled exchangers $A_1, A_2, \ldots, A_n$, with overall parallelflow arrangement (Fig. 1(a)), the total effectiveness P_1 is given by

$$P_1 = \frac{1}{1+R_1}\left\{1 - \prod_{i=1}^{n}\left[1-(1+R_1)P_{1A_i}\right]\right\} \tag{5}$$

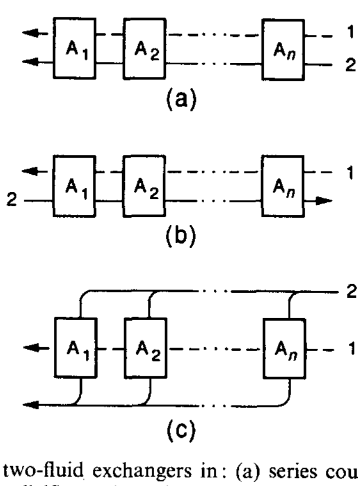


FIG. 1. n two-fluid exchangers in: (a) series coupling with overall parallelflow; (b) series coupling with overall counterflow; (c) parallel coupling.

where

$$R_1 = R_{1A_i}, \quad NTU_1 = \sum_{i=1}^{n} NTU_{1A_i}, \quad i = 1, 2, \ldots, n \tag{6}$$

and P_{1A_i}, NTU_{1A_i} and R_{1A_i} are the effectiveness, number of transfer units and heat capacity rate ratio for the A_i exchanger.

For n series coupled exchangers $A_1, A_2, \ldots, A_n$ with overall counterflow arrangement (Fig. 1(b)), the total effectiveness P_1 is given by

$$P_1 = \frac{\prod_{i=1}^{n}(1-R_1P_{1A_i}) - \prod_{i=1}^{n}(1-P_{1A_i})}{\prod_{i=1}^{n}(1-R_1P_{1A_i}) - R_1\prod_{i=1}^{n}(1-P_{1A_i})} \quad \text{for } R_1 \neq 1 \tag{7a}$$

and

$$P_1 = \frac{\sum_{i=1}^{n}\frac{P_{1A_i}}{1-P_{1A_i}}}{1+\sum_{i=1}^{n}\frac{P_{1A_i}}{1-P_{1A_i}}} \quad \text{for } R = 1 \tag{7b}$$

where

$$R_1 = R_{1A_i}, \quad NTU_1 = \sum_{i=1}^{n} NTU_{1A_i}. \tag{8}$$

For the n parallel coupled exchangers of Fig. 1(c), the exchanger overall effectiveness P_1 is given by

$$P_1 = 1 - \prod_{i=1}^{n}(1-P_{1A_i}) \tag{9}$$

where

$$\frac{1}{R_1} = \sum_{i=1}^{n}\frac{1}{R_{1A_i}}, \quad NTU_1 = \sum_{i=1}^{n} NTU_{1A_i}. \tag{10}$$

An example of a compound assembly. The Domingos method can be applied to many specific compound assemblies of heat exchangers within the limitations of the method to be described later. Consider the case of an n-tube row tube-and-fin single-pass exchanger of Fig. 2. Here we will idealize that Fluid 1 is unmixed over individual finned tubes but it is mixed between tube rows for a conservative magnitude of the exchanger effectiveness. Fluid 2 flows in a finite number of tube rows n; Fluid 2 will be considered mixed in any given tube row, but unmixed from one to another tube row. This exchanger can be modeled as shown in Fig. 3, in which the component exchangers associated with each tube row have identical $NTU_{1,i}$ and they represent a crossflow exchanger with Fluid 1 unmixed and Fluid 2 mixed, and Fluid 1 mixed between units. Thus, this model represents n identical exchangers having Domingos' parallel coupling and we can write down the exchanger overall effectiveness P_1 from equation (9) as

$$P_1 = 1-(1-P_{1,i})^n \tag{11}$$

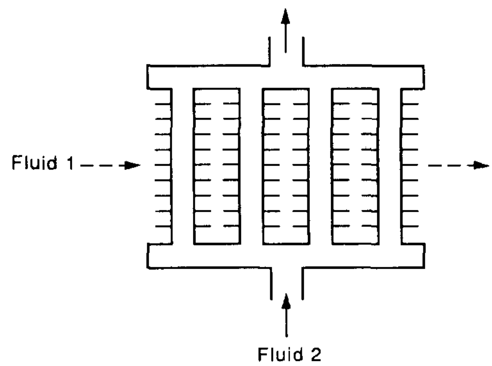


FIG. 2. A single-pass tube-and-fin crossflow exchanger with Fluid 1 unmixed over tubes and mixed between tube rows, and Fluid 2 split into n (4 as shown) equal streams mixed individually in each tube row but not mixed from tube to tube rows.

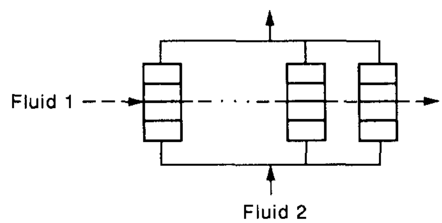


FIG. 3. Modeling of Fig. 2 exchanger.

where all individual exchangers have the same $P_{1,i}$ which is given by Kays and London [20] as

$$P_{1,i} = \frac{1}{nR_1}\{1-\exp[-nR_1(1-\exp\{-NTU_1/n\})]\} \tag{12}$$

where it follows from equation (10) that $R_{1,i} = nR_1$ and $NTU_{1A_i} = NTU_1/n$ and $NTU_1 = UA/C_1$ is the total number of transfer units for the whole exchanger. The exchanger of Fig. 3 and equations (11) and (12) are presented as equation (1.1) in Table 5.

Now if a tube and fin exchanger consists of m passes with each pass containing n tube rows of Fig. 3, the exchanger can be modeled as shown in equation (1.3) or (1.5), in Table 5, depending upon whether the overall connection is parallelflow or counterflow. The whole exchanger consists of m passes with each pass having the same model as in Fig. 3. The temperature effectiveness of this m pass exchanger can be immediately written as shown in Table 5 using the Domingos method for m identical exchangers in a series coupling with an overall parallelflow (equation (5)) or overall counterflow (equation (7)). Here for a given pass, its effectiveness is given by equation (11).

The aforementioned Domingos method is used to obtain the effectiveness of six new flow arrangements, for which we have not found any results reported in the literature. The results are presented in Table 5 as equations (1.1)–(1.6).

Limitations of the Domingos method. An in-depth analysis of the Domingos method indicates that only some complex heat exchanger assemblies can be analyzed with it [18, 21]. These complex assemblies must have the following properties: (1) there are only two inlet and two outlet streams for each individual exchanger, (2) there is a complete mixing of each fluid in the inlet and outlet streams from individual exchangers, and (3) the whole assembly has to be reducible to a single exchanger by successive operations in which pairs of series or parallel coupled exchangers are replaced by single equivalent units. Of these three limitations, the first two deal with the nature of the components to be assembled, and the third regards the ways in which these components are coupled to each other. We will now discuss how to relax the limitations of the Domingos method.

Generalized matrix formalism

The Domingos method is restricted to two inlet and two outlet streams for individual exchangers (or components), and as a result, it involves 2×2 thermal matrices (see equation (4)). If we have m outlet and n inlet streams ($m, n \geqslant 2$), the resulting thermal matrix will be an $m \times n$ matrix. For example, for a 1–1 TEMA J shell-and-tube exchanger of Fig. 4, three outlet streams are related to two inlet streams as follows [22]:

$$\begin{bmatrix} t'_{1,1} \\ t'_{1,2} \\ t'_2 \end{bmatrix} = \begin{bmatrix} 1-P_{1,1} & P_{1,1} \\ 1-P_{1,2} & P_{1,2} \\ P_2 & 1-P_2 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}. \quad (13)$$

Assuming that the two outlet Fluid 1 streams have equal flow rates and using $P_2 = P_1 R_1 = R_1(P_{1,1}+P_{1,2})/2$, the above 3×2 matrix M can be represented as

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \\ M_{31} & M_{32} \end{bmatrix} = \begin{bmatrix} 1-P_{1,1} & P_{1,1} \\ 1-P_{1,2} & P_{1,2} \\ P_2 & 1-P_2 \end{bmatrix} = \begin{bmatrix} 1-P_{1,1} & P_{1,1} \\ 1-P_{1,2} & P_{1,2} \\ P_1 R_1 & 1-P_1 R_1 \end{bmatrix}. \quad (14)$$

Similarly, for m outlet streams and n inlet streams, a generalized $m \times n$ thermal matrix can be written as in ref. [13].

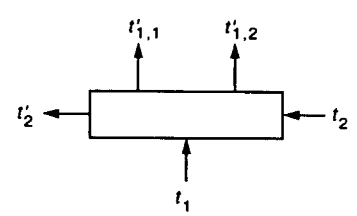


FIG. 4. 1–1TEMA J shell-and-tube exchanger.

The analysis of series or parallel coupling of heat exchangers with multiple streams depends upon how complex individual exchangers are. Some examples of such coupling are presented in refs. [13, 22]. In general, a simpler chain rule method can be used directly for complex flow arrangements having more than two inlet and two outlet streams. In such a case, with the chain rule method, we do not even need to find all the elements of the $m \times n$ thermal matrix (such as that of equation (14)) but we evaluate directly only the necessary elements of the $m \times n$ thermal matrix needed for the determination of the exchanger effectiveness.

Exchanger assemblies with connecting unmixed streams

The second limitation of the Domingos method is that there is a complete mixing of the inlet and outlet streams of the component elements in the assembly. The generalized matrix formalism was extended in ref. [16] to unmixed fluid streams either at inlet, outlet or both at inlet and outlet for either one fluid or both fluids. The continuous temperature distribution of the unmixed stream is represented as a function of a normalized coordinate x (for inlet) or x' (for outlet) as shown in Fig. 5.

The method of analysis for unmixed streams is an extension of the multiple streams at the inlet or outlet of the previous section. The thermal matrix retains its basic character, in as much as it still has the discrete indices that identify the fluid streams, but now it acquires the dependence on one x-type continuous variable for each inlet or outlet unmixed stream present. Whenever the summation over indices occurs in the discrete stream case, an integration over the x-type variable is now present. Let us first derive the thermal matrix for two specific continuous temperature distributions, for illustration, and then generalize the results.

As a first example, we consider the specific crossflow exchanger of Fig. 5(a). Fluid 1 outlet temperature is $t'_1(x')$, and the temperature effectiveness of one fluid stream at any x' is given by

$$p'_1(x') = [t'_1(x') - t_1]/(t_2 - t_1). \quad (15)$$

Solving this equation for $t'_1(x')$ in terms of the inlet temperatures and using equation (2), which is still valid, we can write in matrix notation

$$\begin{bmatrix} t'_1(x') \\ t'_2 \end{bmatrix} = \begin{bmatrix} 1-p'_1(x') & p'_1(x') \\ P_2 & 1-P_2 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} M_{11}(x') & M_{12}(x') \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}. \quad (16)$$

The thermal matrix in this case depends only on the x' variable as follows:

$$M(x') = \begin{bmatrix} 1-p'_1(x') & p'_1(x') \\ P_2 & 1-P_2 \end{bmatrix}. \quad (17)$$

The overall Fluid 1 effectiveness P_1 is defined as the

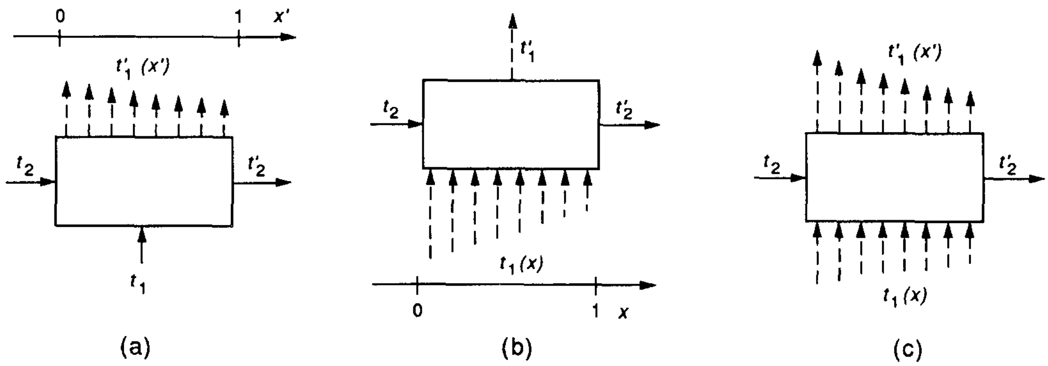


FIG. 5. A single-pass crossflow exchanger with Fluid 2 mixed and Fluid 1 unmixed with: (a) mixed stream at inlet and unmixed streams at outlet; (b) unmixed streams at inlet and the outlet stream mixed at the exchanger outlet; (c) unmixed streams at inlet and outlet.

integrated value of the outlet effectiveness density $p'_1(x')$

$$P_1 = \int_0^1 p'_1(x')\,dx' = \int_0^1 p'_1(x)\,dx \tag{18}$$

where the dummy variable of integration x' has been changed to x.

Similarly, for the case of the continuous temperature distribution at the inlet as shown in Fig. 5(b), it can be shown that the inlet and outlet temperatures are related as follows [16]:

$$\begin{bmatrix} t'_1 \\ t'_2 \end{bmatrix} = \int_0^1 \begin{bmatrix} 1-p_1(x) & P_1 \\ R_1 p_1(x) & 1-R_1P_1 \end{bmatrix} \begin{bmatrix} t_1(x) \\ t_2 \end{bmatrix} dx \tag{19}$$

or

$$M(x) = \begin{bmatrix} 1-p_1(x) & P_1 \\ R_1 p_1(x) & 1-R_1P_1 \end{bmatrix} \tag{20}$$

with

$$P_1 = \int_0^1 p_1(x)\,dx. \tag{21}$$

For the case of Fig. 5(c), the temperatures are related as follows [16]:

$$\begin{bmatrix} t'_1(x') \\ t'_2 \end{bmatrix} = \int_0^1 \begin{bmatrix} M_{11}(x',x) & M_{12}(x') \\ M_{21}(x) & M_{22} \end{bmatrix} \begin{bmatrix} t_1(x) \\ t_2 \end{bmatrix} dx. \tag{22}$$

Here the matrix elements M_{11}, M_{12}, etc. cannot be written explicitly in terms of the exchanger temperature effectiveness, since the outlet temperatures will most probably be a complicated function of the inlet temperatures depending upon the exchanger flow arrangement. For example, consider the case of Fluid 2 mixed and Fluid 1 unmixed of Fig. 5(c), in which we assume the box to represent a single-pass unmixed–mixed crossflow exchanger. The nonuniform outlet temperature distribution $t'_1(x')$ is given in terms of the nonuniform inlet temperature distribution as shown in Table 2 from Braun [23] or Pignotti and Cordero [24]. Comparing this temperature distribution with equation (22), the matrix elements can readily be found as reported in the right-most column of Table 2. Note that Figs. 5(a) and (b) are the special cases of Fig. 5(c). Figure 5(a) represents the inlet temperature as uniform; in Fig. 5(b), the outlet temperature is represented by a mixed mean temperature (by performing an integration of $t'_1(x')$ from $x' = 0$ to 1). Table 2 provides the matrix elements for all three cases of unmixed streams on one fluid side, the other fluid side being mixed.

For the chain rule to be discussed, we will need to interpret the matrix elements as coefficients of inlet temperatures (mixed or unmixed) in the expansion of outlet temperatures (mixed or unmixed). Such a listing is shown in Table 3 for the three cases of Fig. 5, and Table 3 is a counterpart of Table 1 for unmixed streams.

We can generalize the results of equation (22) as follows when two inlet and two outlet streams are unmixed:

$$\begin{bmatrix} t'_1(x') \\ t'_2(x') \end{bmatrix} = \int_0^1 \begin{bmatrix} M_{11}(x',x) & M_{12}(x',x) \\ M_{21}(x',x) & M_{22}(x',x) \end{bmatrix} \begin{bmatrix} t_1(x) \\ t_2(x) \end{bmatrix} dx. \tag{23}$$

The analysis of series and parallel coupling of exchangers with one of the streams connected by continuous temperature distribution has been performed in ref. [16]. No general rules are possible for such couplings except for simple cases. Again, the Pignotti [18] chain rule method is the easiest method for the analysis of the exchangers with continuous temperature distributions, which is discussed next.

THE CHAIN RULE

Domingos [11] outlined rules for evaluating a series or parallel coupled assembly of heat exchangers under

Table 2. Outlet fluid temperature distributions and the thermal matrix elements for the configurations of Fig. 5

Geometry	Temperature distributions	Outlet/inlet thermal matrix elements
Fig. 5(a)	$t_1'(x') = (1-K e^{-\lambda x'})t_1 + K e^{-\lambda x'} t_2$	$M_{11}(x') = 1-p_1'(x')$
	$t_2' = (1-e^{-\lambda})t_1 + e^{-\lambda} t_2$	$M_{12}(x') = p_1'(x')$
	$p_1'(x') = K e^{-\lambda x'}$	$M_{21} = P_2 = P_1 R_1$
	$P_1 = (1-e^{-\lambda})/R_1$	$M_{22} = 1-P_2 = 1-P_1 R_1$
Fig. 5(b)	$t_1' = \int_0^1 \{1-K\exp[-\lambda(1-x)]\}$	$M_{11}(x) = 1-p_1(x)$
	$\times t_1(x)\,dx + (1-e^{-\lambda})t_2/R_1$	$M_{12}(x) = P_1$
	$t_2' = \int_0^1 KR_1 \exp[-\lambda(1-x)]$	$M_{21}(x) = R_1 p_1(x)$
	$\times t_1(x)\,dx + e^{-\lambda} t_2$	
	$p_1(x) = K\exp[-\lambda(1-x)]$	$M_{22} = 1-P_1 R_1$
	$P_1 = (1-e^{-\lambda})/R_1$	
Fig. 5(c)	$t_1'(x') = (1-K)t_1(x')$	$M_{11}(x',x) = (1-K)\delta(x'-x)$
	$+\lambda K e^{-\lambda x'} \int_0^{x'} e^{\lambda x} t_1(x)\,dx$	$+\lambda K \exp[-\lambda(x'-x)]H(x'-x)$
	$+K\exp(-\lambda x')t_2$	$M_{12}(x') = K\exp(-\lambda x')$
	$t_2' = \lambda e^{-\lambda} \int_0^1 e^{\lambda x} t_1(x)\,dx$	$M_{21}(x) = \lambda \exp[-\lambda(1-x)]$
	$+e^{-\lambda} t_2$	$M_{22} = \exp(-\lambda)$
	where $t_1(x') = t_1(x)$	$\int_a^b \delta(x'-x) f(x)\,dx = f(x')$
	at $x = x'$	for $a < x < b$
		$H(x'-x) = \begin{cases} 1 & \text{for } x' > x \\ 0 & \text{for } x' < x \end{cases}$

In all above, $\lambda = KR_1$, $K = 1-\exp(-NTU_1)$. Here NTU_1 and R_1 are for the single crossflow unit shown in Figs. 5(a)–(c).

Table 3. Matrix elements for combinations of inlet and outlet mixed/unmixed streams of Fig. 5

Table 3(a) for Fig. 5(a)

	Mixed Fluid 1 inlet	Mixed Fluid 2 inlet
Unmixed Fluid 1 outlet	$1-p_1'(x')$	$p_1'(x')$
Mixed Fluid 2 outlet	P_2	$1-P_2$

Table 3(b) for Fig. 5(b)

	Unmixed Fluid 1 inlet	Mixed Fluid 2 inlet
Mixed Fluid 1 outlet	$1-p_1(x)$	P_1
Mixed Fluid 2 outlet	$R_1 p_1(x)$	$1-R_1 P_1$

Table 3(c) for Fig. 5(c)

	Unmixed Fluid 1 inlet	Mixed Fluid 2 inlet
Unmixed Fluid 1 outlet	$M_{11}(x',x)$	$M_{12}(x')$
Mixed Fluid 2 outlet	$M_{21}(x)$	M_{22}

the limitations of (1) only two inlet and two outlet streams for each individual exchanger, and (2) a complete mixing of each fluid in the inlet and outlet streams from individual exchangers. In the preceding two sections, we discussed how to take into account the above two limitations. Now we will discuss how to evaluate the exchanger effectiveness when exchangers have either multiple (more than two) inlet and/or outlet streams, unmixed streams between exchangers or passes, and the assembly may or may not be coupled only in series or parallel coupling (i.e. connected by some compound coupling).

A general rule can be given for the calculation of the effectiveness of the compound assemblies of heat exchangers with or without multiple inlet/outlet streams and unmixed streams. Before formulating it, we need to introduce a very simple extension of the matrix formalism.

Dividers and mixers

The node S in Fig. 6(c) is referred to as a divider where one stream is divided into two (and in general m) streams. As there is only one inlet stream, the temperatures of outlet streams are equal to the inlet one; and the thermal matrix has a single column with matrix elements equal to one. In the example under consideration for node S, we have

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} [t_{13}] \quad \text{or} \quad S = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad (24)$$

where we use the letter S to designate the thermal matrix (in this case a column matrix) associated to node S. If there is more than one inlet stream and only one outlet stream at a node such as Q in Fig. 6(c), it is referred to as a mixer. The temperature of the mixed outlet stream is the weighted average of the inlet temperatures, and the thermal matrix has just a single row. In the example under consideration, for equal flow splits in branches 7 and 8, we simply have

$$t_9 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} t_7 \\ t_8 \end{bmatrix} \quad \text{or} \quad Q = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix}. \qquad (25)$$

Here Q is the (row) thermal matrix associated to node Q. Observe that it is still true that the sum of the matrix elements in each row is equal to unity.

Interpretation of matrix elements

In the chain rule, we will use specific matrix elements of component exchangers. If the component exchanger is a 2×2 matrix, the interpretation of a specific element M_{ij} in terms of exchanger properties is shown in Table 1. Here the subscript i identifies the Fluid 1 or Fluid 2 outlet stream and similarly the subscript j identifies the Fluid 1 or Fluid 2 inlet stream. If the component exchanger has multiple inlet and/or outlet streams, a counterpart of Table 1 needs to be generated and interpreted in the same way. For example, for the 1–1 TEMA J shell-and-tube exchanger of Fig. 4, a counterpart of Table 1, can be generated from equation (14) and subsequently the value of any element M_{ij} can be found immediately from the inspection of that table. If the component exchanger has unmixed Fluid 1 streams at the inlet or outlet and Fluid 2 mixed, the expression for the specific matrix element M_{ij} can be found from Table 3.

The chain rule

Consider an assembly of heat exchangers Z, Y, $W, \ldots, A$ that are connected in an arbitrary (but specified) order. Let the overall assembly, designated as M, be a two-fluid exchanger having two inlet fluid streams and two outlet fluid streams. The overall effectiveness of this assembly can be determined easily from equation (4) or Table 1 if any one element M_{ij} is known. The chain rule offers a scheme to relate an assembly element M_{ij} to individual component elements Z_{ij}, Y_{ij}, etc. as follows:

$$M_{ij} = \sum_{\text{paths}} Z_{iz} Y_{zy} W_{yw} \cdots B_{cb} A_{bj}. \qquad (26)$$

The summation has to be performed over all possible paths that link the inlet stream j to the outlet stream i, following all combinations of streams and exchangers, and always in the direction of flow indicated by the arrows. Each path contributes a term that is equal to the product of all matrix elements linking the stream temperatures found along the path. Observe that along each one of these paths, the inlet stream for one exchanger is the outlet stream of the preceding one; hence, the indices are 'chained' in equation (26), which is typical of matrix multiplication.

Equation (26) can be derived by writing the linear relationship between outlet and inlet temperatures for each of the components, and equating the temperatures of streams shared by pairs of exchangers. Even though for concreteness in the above expression, M was assumed to represent an exchanger with two inlet and two outlet streams, the same expression holds for an arbitrary number of streams.

Methodology for determining exchanger effectiveness using the chain rule

We will now outline the steps of this methodology using an example of 1–1 TEMA H shell-and-tube exchanger as shown in Fig. 6 with its modeling. We will derive its effectiveness P_1 by evaluating the matrix element $M_{9,13}$ $(=P_1)$ of the model of Fig. 6(b).

(1) Decompose a given exchanger or an assembly into smaller elements representing simple 'known' exchangers whenever possible. This is done in Fig. 6(b) for the 1–1 TEMA H exchanger. Determine the heat capacity rates for each fluid stream going into each elemental exchanger and the surface area associated with each elemental exchanger. Subsequently determine $R_{1,j}$ and $NTU_{1,j}$ for each elemental exchanger.

(2) Identify all elemental exchangers (A, $B, \ldots$) and nodes (K, $L, \ldots$) by alphabetic characters and label all streams (1–14) as is done in Fig. 6(b).

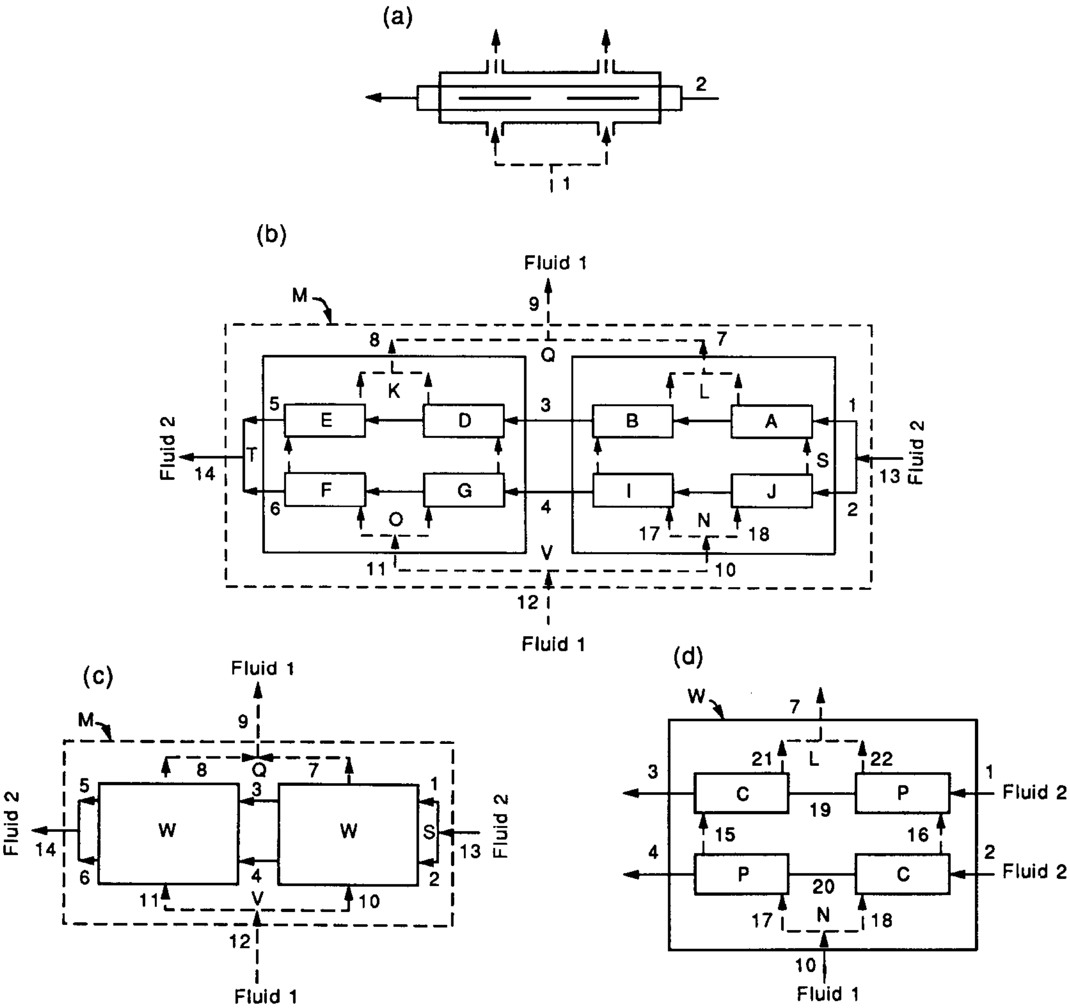


FIG. 6. (a) 1–1 TEMA H shell-and-tube exchanger; (b) detailed modeling of (a); (c) gross modeling of (a); (d) detailed modeling of one-half of (b).

(3) Find all different paths going from stream 13 to stream 9 (notice 13 and 9 are the subscripts of $M_{9,13}$ in reverse order). Along these paths, either the Fluid 1 or Fluid 2 stream or any combination thereof may be followed, but always in the direction of the fluid flow indicated by the arrows.

(4) List these paths in the reverse order.

(5) The value of the matrix element $M_{9,13}$ of interest is then given by (a) first a multiplication of the matrix elements of the exchangers (or nodes) connecting streams for each path identified in step 3 above, and then (b) adding up the individual results of all paths of the preceding step (a) under step 5.

(6) The values of matrix elements of elemental exchangers for the $M_{9,13}$ expression of the preceding step are obtained using Table 1 knowing the outlet and inlet fluid streams identification from subscripts. The values of the nodes are obtained using equations (24) and (25) or similar equations as appropriate.

(7) After determining the values of all elements on the right-hand side of an equation from $M_{9,13}$ of step 5, substitute their values in that equation, and obtain the value of $M_{9,13}$ from Table 1. Simplification of the resulting expression will yield the desired value for P_1.

An example of a multiple stream exchanger using the chain rule

Let us illustrate the above methodology for obtaining P_1 for 1–1 TEMA H shell-and-tube exchanger as shown in Fig. 6(a). This exchanger is modeled as shown in Fig. 6(b) in which it consists of eight exchanger components, four in counterflow (B, E, G, J) and four in parallelflow (A, D, F, I). This exchanger can be analyzed using the detailed modeling of Fig. 6(b). However, in order to simplify the algebra, we will consider the geometrical (and *not* performance) similarity of two halves, modeled as two W units in Fig. 6(c), and analyze them first. Next, we provide all necessary identification to the right-most W as shown in Fig. 6(d), designate the component exchangers by C (counterflow) and P (parallelflow). Now let us follow the steps of the chain rule methodology.

(1) Fluid 1 at inlet (stream 12) divides into two equal streams 10 and 11 (see Fig. 6(b)) and each stream further divides equally into two (such as 10 into 17 and 18 as shown in Fig. 6(d)). Thus for each component exchanger, $C_{1A} = C_{1B} = \cdots = C_1/4$. Fluid 2 at the inlet (stream 13) divides into two parallel streams 1 and 2, and each stream flows through four component exchangers. Hence, for each component exchanger, $C_{2A} = C_{2B} = \cdots = C_2/2$. Therefore

$$R_{1A} = R_{1B} = \cdots = R_{1J} = \frac{C_1/4}{C_2/2} = \frac{R_1}{2} \quad (27)$$

where $R_1 = C_1/C_2$ is the heat capacity rate ratio for the whole exchanger.

Each component exchanger has 1/8 of the total exchanger area and as above $C_{1A} = C_{1B} = \cdots = C_1/4$. Idealizing constant overall heat transfer coefficient U, we have

$$NTU_{1A} = NTU_{1B} = \cdots = NTU_{1J} = \frac{UA/8}{C_1/4} = \frac{1}{2} NTU_1 \quad (28)$$

where $NTU_1 = UA/C_1$ is the number of transfer units for the whole exchanger.

(2) All component exchangers, streams and nodes have been identified in the models of Figs. 6(b)–(d).

Now let us analyze the assembly of Fig. 6(c) first. The overall exchanger is represented by M which has two identical components W. Let us evaluate the matrix element $M_{9,13}$ which relates Fluid 1 outlet to Fluid 2 inlet and hence $M_{9,13} = P_1$ using Table 1 as a guide. Now continue to follow the steps outlined earlier for the chain rule methodology.

(3) From Fig. 6(c), there are six paths leading from stream 13 to 9 as follows: (1) 13–1–7–9, (2) 13–2–7–9, (3) 13–1–3–8–9, (4) 13–2–3–8–9, (5) 13–1–4–8–9, and (6) 13–2–4–8–9. Note that we have followed streams of Fluids 1 and 2 in all combinations in the forward direction from stream 13 to 9.

(4) Indicate these paths in a reverse order: 9–7–1–13, 9–7–2–13, 9–8–3–1–13, 9–8–3–2–13, 9–8–4–1–13, and 9–8–4–2–13.

(5) Now apply the chain rule for each path, and $M_{9,13}$ is the sum of the results for each path

$$\begin{aligned} M_{9,13} &= Q_{97}W_{71}S_{1,13} + Q_{97}W_{72}S_{2,13} \\ &+ Q_{98}W_{83}W_{31}S_{1,13} + Q_{98}W_{83}W_{32}S_{2,13} \\ &+ Q_{98}W_{84}W_{41}S_{1,13} + Q_{98}W_{84}W_{42}S_{2,13}. \quad (29) \end{aligned}$$

Note that all terms on the right-hand side of equation (29) have subscripts such that any two neighboring subscripts for a fluid stream are the same. If we conceptually cancel these pairs of neighboring subscripts, the only two subscripts that remain for each path are the same as those for M on the left-hand side of equation (29), i.e. 9 and 13.

As noted in equations (25) and (24), $Q_{97} = Q_{98} = 1/2$, and $S_{1,13} = S_{2,13} = 1$. Hence, equation (29) reduces to

$$\begin{aligned} M_{9,13} &= \tfrac{1}{2}(W_{71} + W_{72}) + \tfrac{1}{2}[W_{83}(W_{31} + W_{32}) \\ &+ W_{84}(W_{41} + W_{42})]. \quad (30) \end{aligned}$$

Due to the identical geometries of the two values of W, the corresponding matrix elements are identical (they depend only upon the values of P, i.e. NTU_1 and R_1 only), so

$$W_{83} = W_{71} \quad \text{and} \quad W_{84} = W_{72}. \quad (31)$$

Hence

$$\begin{aligned} M_{9,13} &= \tfrac{1}{2}W_{71}(1 + W_{31} + W_{32}) \\ &+ \tfrac{1}{2}W_{72}(1 + W_{41} + W_{42}). \quad (32) \end{aligned}$$

The thermal matrix for the subassembly W of Fig. 6(c) is a 3×3 matrix because of three inlet (1, 2 and 10) and three outlet (3, 4 and 7) streams. If we want to analyze by the general matrix formalism method, we need to know nine matrix elements. However, the right-hand side of equation (32) indicates that we need to calculate only six elements and will use the chain rule for their determination using the modeling of Fig. 6(d).

(6) Now we will determine the required W_{ij} elements using the modeling of Fig. 6(d). Here individual component exchangers are designated as P (for A, I) for parallelflow and C (for B, J) for counterflow. For the element W_{31}, there is only one path 3–19–1 written in the reverse order for the stream 1 to 3. Therefore

$$W_{31} = C_{3,19}P_{19,1}. \quad (33)$$

Similarly, we can identify individual paths for evaluating the rest of five values of W of equation (32) and write down the following formulas:

$$\begin{aligned} W_{32} &= C_{3,19}P_{19,16}C_{16,2} + C_{3,15}P_{15,20}C_{20,2} \\ W_{71} &= L_{7,22}P_{22,1} + L_{7,21}C_{21,19}P_{19,1} \\ &= \tfrac{1}{2}[P_{22,1} + C_{21,19}P_{19,1}] \\ W_{72} &= L_{7,22}P_{22,16}C_{16,2} + L_{7,21}C_{21,19}P_{19,16}C_{16,2} \\ &+ L_{7,21}C_{21,15}P_{15,20}C_{20,2} \\ &= \tfrac{1}{2}[P_{22,16}C_{16,2} + C_{21,19}P_{19,16}C_{16,2} \\ &+ C_{21,15}P_{15,20}C_{20,2}] \\ W_{42} &= W_{31} \\ W_{41} &= 0 \text{ since there is no path in the forward} \\ &\quad \text{direction from stream 1 to stream 4.} \quad (34) \end{aligned}$$

(7) The magnitudes of P_{ij} and C_{ij} elements of the preceding step are obtained using Table 1 noting that the subscript i stands for the outlet stream and j for the inlet stream. We will first denote the parallelflow

and counterflow effectivenesses as

$$A = P_{1p} = P_{1p}(NTU_{1p}, R_{1p}) = P_{1p}\left(\frac{NTU_1}{2}, \frac{R_1}{2}\right) \quad (35)$$

$$B = P_{1c} = P_{1c}(NTU_{1c}, R_{1c}) = P_{1c}\left(\frac{NTU_1}{2}, \frac{R_1}{2}\right) \quad (36)$$

where A and B are explicitly given in equation (2.11) of Table 5. Now using Table 1 as a guide, P_{ij} and C_{ij} elements for equation (34) are given by

$$C_{3,19} = C_{20,2} = 1 - R_{1c}P_{1c} = 1 - \frac{R_1}{2}B$$

$$C_{16,2} = C_{21,19} = P_{1c} = B$$

$$C_{3,15} = R_{1c}P_{1c} = \frac{R_1}{2}B$$

$$C_{21,15} = 1 - P_{1c} = 1 - B$$

$$P_{19,16} = R_{1p}P_{1p} = \frac{R_1}{2}A$$

$$P_{15,20} = P_{22,1} = P_{1p} = A$$

$$P_{19,1} = P_{4,20} = 1 - R_{1p}P_{1p} = 1 - \frac{R_1}{2}A$$

$$P_{22,16} = 1 - P_{1p} = 1 - A. \quad (37)$$

(8) Substituting the values of the individual matrix elements from equation (37) into equations (33) and (34) and subsequent values of W into equation (32), and after algebraic manipulations, we get

$$M_{9,13} = P_1 = E[1 + (1 - BR_1/2)(1 - BR_1/2 + ABR_1)] - AB(1 - BR_1/2) \quad (38)$$

where the parameters A, B and E are defined in equation (2.11) of Table 5. Thus, one can see that a complicated flow configuration like the 1–1 H shell-and-tube exchanger can readily be analyzed using the chain rule. The derivation could have been much more complicated if we had used the generalized matrix formalism of ref. [13] which represents an extension to the Domingos method.

An example of a continuous stream between passes using the chain rule

We will analyze a single-pass crossflow exchanger with three tube rows as shown in Fig. 7; Fluid 1 unmixed across each tube row and remains unmixed but twisted between the tube rows, and Fluid 2 is mixed in each tube, but unmixed from one to another tube row. The twisted order for Fluid 1 between the tube rows 1 and 2 means that the fluid stream originating at a position D after the first tube row ends up at a position E on the second tube row, etc. While such a twisted order is of academic interest for a single-pass exchanger, it is of practical interest if U tubes of the same bend radius are used in an exchanger. The solution to this problem is chosen (i) to illustrate the methodology on how to use the chain rule for the analysis of assemblies with intermediate unmixed streams, and (ii) to show how to handle twisted order geometries.

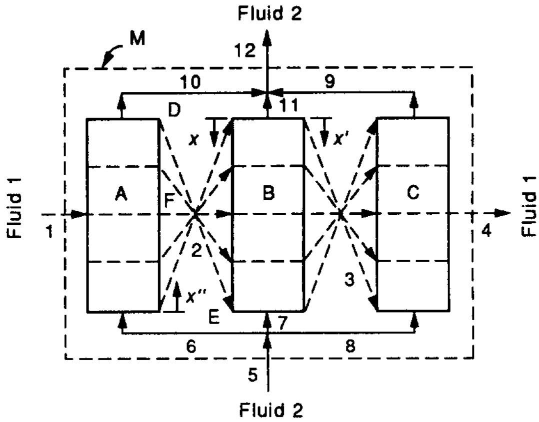


FIG. 7. Modeling of a single-pass crossflow exchanger with Fluid 1 unmixed and twisted and Fluid 2 split into three equal streams individually mixed.

Let us follow the methodology of the chain rule. For each component exchanger, it can be shown that $R_{1A} = R_{1B} = R_{1C} = 3R_1$ and $NTU_{1A} = NTU_{1B} = NTU_{1C} = NTU_1/3$. All component exchangers and streams are identified in Fig. 7. The continuous streams between the tube rows are identified simply by streams 2 and 3, each of which can be considered to be constituted by infinitely many elementary streams, labeled by a variable that ranges between 0 and 1. Before we discuss all paths, let us clarify the variables for identifying stream location at the inlet and outlet of each exchanger A, B and C. Let us designate the variable x for the inlet unmixed streams across the height of the component exchanger B and x' the variable for the outlet unmixed streams from the exchanger B across its height. Both x and x' vary from 0 to 1. Due to the twisting of the streams between tube rows, the inlet stream to exchanger B at a location x is the outlet stream of A at the location $1-x$. Instead we could have started a new variable x'' from the bottom of exchanger A as shown in Fig. 7. In that case the stream at x'' from A will meet B at $x = x''$. But that would require using different coordinate conventions in evaluating the matrix elements of A and B. We preferred instead to use a uniform convention and introduce the twisting effect by assigning the argument $1-x$ to the A matrix element. Similarly, the variable across the height for the inlet fluid streams to the exchanger C will be $1-x'$. For this reason, the variables on which A_{21}, B_{32} and C_{43} matrix elements for individual streams are dependent are shown in parentheses: $A_{21}(1-x)$, $B_{32}(x', x)$ and $C_{43}(1-x')$.

A careful look at the geometry indicates that if we choose to evaluate the matrix element M_{41} for the whole exchanger, the only paths from stream 1 to 4 are infinitely many combinations of paths through continuous streams between A and B, and B and C. For example, let us follow individual stream F between A and B; then after exchanger B, we have an infinite number of paths from B to C that can lead to stream 4. Hence, *one* matrix element for M_{41} will correspond to a multiplication of the matrix element through stream F with matrix elements for infinitely many streams between B and C. Hence, it will be

$$\left\{\int_0^1 C_{43}(1-x')B_{32}(x',x)\,\mathrm{d}x'\right\}A_{21}(1-x). \quad (39)$$

Since there are an infinite number of paths originating from A to B (similar to stream F), the matrix element M_{41} will be given by an integration of equation (39) for all values of x

$$M_{41} = \int_0^1\left\{\int_0^1 C_{43}(1-x')B_{32}(x',x)\,\mathrm{d}x'\right\} \times A_{21}(1-x)\,\mathrm{d}x. \quad (40)$$

Now component exchangers A, B and C can be modeled as shown in Figs. 5(a), (c) and (b), respectively, and their matrix elements can be identified using Table 3. The matrix element $C_{43}(x') = 1-p_1(x')$ from Table 3(b) is also equal to $1-K\exp[-\lambda(1-x')]$ from Table 2 with $K = 1-\exp(-NTU_1/3)$ and $\lambda = 3KR_1$. Since C_{43} here is dependent upon a dummy variable x', the x variable for $p_1(x)$ of Tables 3(b) and 2 is changed to x'. However, we need C_{43} dependent upon $1-x'$ for equation (40), so replacing x' by $1-x'$, we have

$$C_{43}(1-x') = 1-K\exp(-\lambda x'). \quad (41)$$

Similarly, using Tables 3(c) and 2 as a guide, we have

$$B_{32}(x',x) = (1-K)\delta(x'-x) + \lambda K\exp[-\lambda(x'-x)]H(x'-x). \quad (42)$$

And using Tables 3(a) and 2 as a guide

$$A_{21}(x) = 1-p_1'(x) = 1-K\exp(-\lambda x)$$

so

$$A_{21}(1-x) = 1-K\exp[-\lambda(1-x)]. \quad (43)$$

Now equation (40) is first arranged as follows:

$$M_{41} = \int_0^1\int_0^1 C_{43}(1-x')B_{32}(x',x)A_{21}(1-x)\,\mathrm{d}x'\,\mathrm{d}x$$
$$= \int_0^1 C_{43}(1-x')\left[\int_0^1 B_{32}(x',x)A_{21}(1-x)\,\mathrm{d}x\right]\mathrm{d}x'. \quad (44)$$

Substitute the values of $B_{32}(x', x)$ and $A_{21}(1-x)$ from equations (42) and (43), and first carry out the integration of the [ ] term of equation (44), and then substitute the value of $C_{43}(1-x')$ from equation (41) in the resultant equation and perform the second integration. The result is

$$M_{41} = 1-\frac{K}{2}[6-K-K\,\mathrm{e}^{-\lambda}]\,C_0(\lambda) + K^2\left[1-\frac{K}{2}\mathrm{e}^{-\lambda}\right]C_0(2\lambda)+\tfrac{1}{2}(2-K)K^2\,\mathrm{e}^{-\lambda}. \quad (45)$$

Note that $M_{41} = 1-P_1$. So we can readily obtain the value of P_1 from equation (45). After some algebraic manipulations, we obtain the resultant expression for P_1 as shown in Table 5 for equation (2.3).

Loops

In the two examples outlined for the use of the chain rule, there were a finite number of paths that led from the inlet to the outlet stream selected. However, in some cases, there may be infinitely many paths from the inlet to the outlet stream selected. For example, consider the series coupled exchanger A and B in overall counterflow of Fig. 8. If we choose to determine the M_{52} element, the following paths are possible from stream 2 to 5 written in a reverse order: (1) 5–2, (2) 5–3–4–2, (3) 5–3–4–3–4–2, (4) 5–3–4–3–4–3–4–2, etc. We conclude that there are infinitely many paths because of a loop between A and B; we can generally denote these paths as $5\text{–}(3\text{–}4)^n-2$ with $n = 0, 1, 2, 3, \ldots$. The corresponding contributions to the matrix elements are

$$M_{52} = A_{52}+\cdots+A_{53}B_{34}[A_{43}B_{34}]^{(n-1)}A_{42}+\cdots, \quad n = 1, 2, \ldots. \quad (46)$$

Remembering the expansion

$$1/(1-x) = 1+x+x^2+x^3+\cdots \quad (47)$$

equation (46) simplifies to

$$M_{52} = A_{52}+A_{53}B_{34}A_{42}/(1-A_{43}B_{34}). \quad (48)$$

Subsequent substitution of the values of the matrix elements on the right-hand side of this equation will yield the value of $M_{52} = P_1$ for the exchanger as equation (7) with $n = 2$.

More complicated configurations may involve single or multiple loops that are more difficult to handle explicitly. Using the chain rule, Pignotti [18] has devised a method to analyze exchanger configurations with single or multiple loops. The method involves cutting suitably chosen internal flow lines in the assembly to eliminate the loops, such as stream 3 or 4 (only one) in Fig. 8, applying the chain rule to the resultant geometry, and writing the original matrix elements in terms of those of the cut graphs. As long as intermediate streams that form loops are mixed,

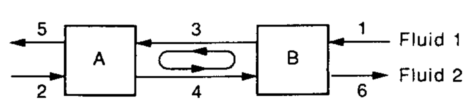


FIG. 8. Series coupled exchangers A and B in overall counterflow with all fluid streams identified.

Table 4. The expected results using the chain rules for exchanger assemblies with mixed or unmixed streams between exchangers and with or without loops

Intermediate streams	Loops	Other conditions	Result
Mixed	No	None	Algebraic
	Yes	None	Algebraic
Unmixed	No	None	Analytic (integral)
	Yes	Cutting mixed streams eliminates all loops	Analytic (integral)
		Requires cutting unmixed streams to eliminate all loops	Numerical

the procedure outlined in ref. [18] will yield closed-form algebraic solution for P_1. Other limitations with multiple loops are summarized next.

Limitations of the chain rule

The chain rule that is described represents a powerful tool to obtain the effectiveness of many highly complicated exchanger assemblies. Whenever all the intervening streams in an exchanger assembly are mixed, an algebraic closed-form expression for P_1 in terms of the matrix elements of the component exchangers can *always* be obtained, although the algebra may be difficult and time consuming, and the final expression may be very long and complicated depending upon the assembly. When there are intermediate unmixed streams between exchangers and there are no loops, again the closed-form expression for the effectiveness of the assembly is possible, and it is analytic which means that the result can be expressed in an integral form. With loops and intermediate unmixed streams, if cutting of mixed streams (somewhere in the assembly) eliminates all loops, the resultant expression for P_1 can, again, be expressed as analytic. Finally, if the cutting of unmixed streams is required to eliminate all loops, the exchanger effectiveness can only be obtained numerically at present. Table 4 summarizes the power of the chain rule and the type of expected results.

RULES FOR EXCHANGERS WITH ONE FLUID MIXED

Consider a multipass exchanger having at least *one* fluid side perfectly mixed throughout, the other side being mixed, unmixed or partially mixed. If the effectiveness of such an exchanger is known for overall parallelflow (or counterflow), its effectiveness for overall counterflow (or parallelflow) can be given by the rules outlined in ref. [17]. The effectiveness P_1 of the original exchanger and the effectiveness $\hat{P}_1$ of the exchanger with one fluid reversed are related as follows:

$$\hat{P}_1(R_1, NTU_1) = P_1(-R_1, NTU_1)/[1+R_1P_1(-R_1, NTU_1)]. \quad (49)$$

Here, the subscript 1 refers to the fluid side having no restrictions (i.e. it can be mixed, unmixed or split). The temperature effectiveness of the mixed fluid is then given by

$$\hat{P}_2(R_2, NTU_2) = -P_2(-R_2, -NTU_2)/[1-P_2(-R_2, -NTU_2)]. \quad (50)$$

It must be emphasized that equation (49) or (50) is a mathematical relationship between the values of P of the original and inverted exchangers. $\hat{P}$ of the inverted exchanger for the physical (positive) values of R is related to P of the original exchanger for the unphysical (negative) values of R as shown in equations (49) and (50).

Using these rules, the effectivenesses of three new exchanger configurations have been obtained and included as equations (2.2), (2.10) and (2.12) in Table 5.

NEW RESULTS

A number of new exchanger configurations have been analyzed by the aforementioned three rules: (1) the Domingos rules, (2) Pignotti chain rules and (3) rules for exchangers with one fluid mixed. They are briefly summarized next.

Results using the Domingos rules

A single-pass crossflow exchanger with a finite number of tube rows has been analyzed extensively for the case of out-of-tube fluid unmixed and the tube fluid mixed in each tube row (such as in Fig. 2) [23–28]. However, when the out-of-tube fluid is either mixed between tube rows or is mixed throughout, no results for the exchanger effectiveness are available, and have been presently derived using the Domingos rules and summarized as equations (1.1) and (1.2) in Table 5. When m passes of such exchangers are connected in overall parallelflow or counterflow, the overall exchanger effectiveness can again be obtained by using the Domingos rules. They are summarized as equations (1.3)–(1.6) in Table 5.

Table 5. P_1–NTU_1 formulas for some complex heat exchanger flow arrangements derived by present authors

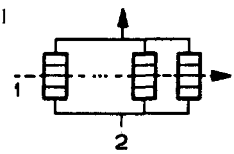
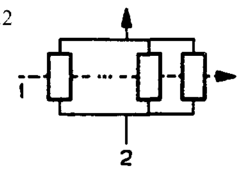
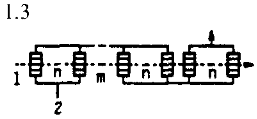
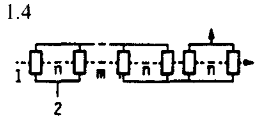
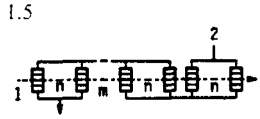
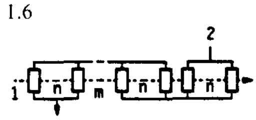
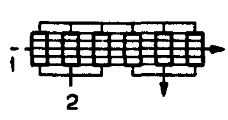
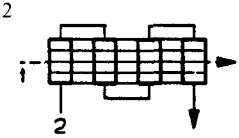
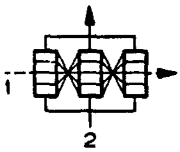
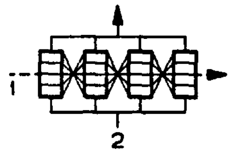
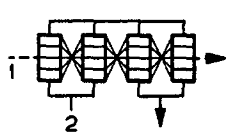
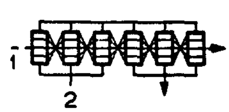
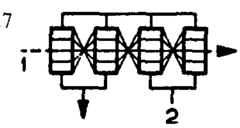
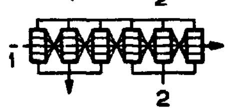
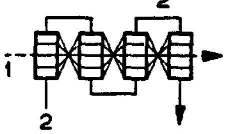
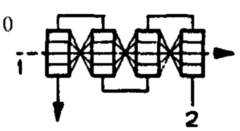
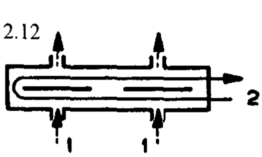
Equation no.	Sketch of flow arrangement	Description of flow arrangement	P_1–NTU_1–R_1 formulas
1.1		Single-pass crossflow exchanger; Fluid 2 split into n equal streams individually mixed; Fluid 1 mixed between Fluid 2 streams, otherwise unmixed.	$P_1 = 1-(1-A)^n$ $A = [1-\exp(-nKR_1)]/(nR_1)$ $K = 1-\exp(-NTU_1/n)$ For large n, this approaches the single-pass crossflow exchanger, Fluid 1 mixed, Fluid 2 unmixed.
1.2		Single-pass crossflow exchanger; Fluid 2 split into n streams individually mixed; Fluid 1 mixed throughout.	$P_1 = 1-(1-A)^n$ $A = [1/K_1+nR_1/K_2-n/NTU_1]^{-1}$ $K_1 = 1-\exp(-NTU_1/n)$; $K_2 = 1-\exp(-R_1 NTU_1)$ For large n, this approaches the single-pass crossflow exchanger, Fluid 1 mixed, Fluid 2 unmixed.
1.3		m-pass cross-parallelflow exchanger; Fluid 2 mixed between passes, and within each pass, split into n streams individually mixed; Fluid 1 mixed between Fluid 2 streams, otherwise unmixed.	$P_1 = \{1-[1-(1+R_1)B]^m\}/(1+R_1)$ $B = 1-(1-A)^n$ $A = [1-\exp(-nKR_1)]/nR_1$ $K = 1-\exp(-NTU_1/nm)$
1.4		m-pass cross-parallelflow exchanger; Fluid 2 mixed between passes, and within each pass, split into n streams individually mixed; Fluid 1 mixed throughout.	$P_1 = \{1-[1-(1+R_1)B]^m\}/(1+R_1)$ $B = 1-(1-A)^n$; $A = [1/K_1+nR_1/K_2-nm/NTU_1]^{-1}$ $K_1 = 1-\exp(-NTU_1/nm)$ $K_2 = 1-\exp(-R_1 NTU_1/m)$
1.5		m-pass cross-counterflow exchanger; Fluid 2 mixed between passes, and within each pass, split into n streams individually mixed; Fluid 1 mixed between Fluid 2 streams, otherwise unmixed.	$P_1 = [(1-R_1B)^m-(1-B)^m]/[(1-R_1B)^m-R_1(1-B)^m]$ $B = 1-(1-A)^n$; $A = [1-\exp(-nKR_1)]/nR_1$ $K = 1-\exp(-NTU_1/nm)$
1.6		m-pass cross-counterflow exchanger; Fluid 2 mixed between passes, and within each pass, split into n streams individually mixed; Fluid 1 mixed throughout.	$P_1 = [(1-R_1B)^m-(1-B)^m]/[(1-R_1B)^m-R_1(1-B)^m]$ $B = 1-(1-A)^n$; $A = [1/K_1+nR_1/K_2-nm/NTU_1]^{-1}$ $K_1 = 1-\exp(-NTU_1/nm)$ $K_2 = 1-\exp(-R_1 NTU_1/m)$
2.1		Two-pass cross-parallelflow exchanger; Fluid 1 unmixed and planar; Fluid 2 mixed between passes. Within each pass, Fluid 2 is split into three streams individually mixed. This configuration represents a 2-pass air cooler, 3 rows per pass, overall parallel connection, air unmixed throughout.	$P_1 = 2A-R_1A^2-\delta$ $A = a_0C_0(3R_1K)+a_1C_1(3R_1K)+a_2C_2(3R_1K)$ $a_0 = 1-(1-K)^3$; $a_1 = 3R_1K^3(3-2K)$; $a_2 = 9R_1^2K^5/2$ $\delta = a_0^2C_0(6R_1K)+2a_0a_1C_1(6R_1K)+(a_1^2+2a_0a_2)C_2(6R_1K)$ $+2a_1a_2C_3(6R_1K)+a_2^2C_4(6R_1K)$ $C_n(z) = n![1-\exp(-z)(1+z+\cdots+z^n/n!)]/z^{n+1}$ $K = 1-\exp(-NTU_1/6)$
2.2		Four-pass cross-parallelflow exchanger, Fluid 1 unmixed and planar, Fluid 2 mixed. It represents a 4-pass air cooler with 1 row per pass.	$P_1 = (2A-A^2-B)/R_1$ $A = (1-K/2)(1-\alpha)$; $B = K(1-K/2)^2(1+4KR_1\alpha-\alpha^2)/2$ $K = 1-\exp(-NTU_1/4)$; $\alpha = \exp(-2KR_1)$

Table 5.—*continued*

Equation no.	Sketch of flow arrangement	Description of flow arrangement	P_1–NTU_1–R_1 formulas
2.3		Single-pass crossflow exchanger; Fluid 1 unmixed and twisted; Fluid 2 is split into three equal streams individually mixed.	$P_1 = (a_0+b_0)C_0(3KR_1)+a_1C_1(3KR_1)$ $a_0 = K\{(2-K)[1-(K/2)\exp(-3KR_1)]+(K^2/4)\exp(-6KR_1)\}$ $b_0 = (1-K)(1-K/2)K+(K^2/2)[1-(K/2)\exp(-3KR_1)]$ $a_1 = 3K^3R_1(1-K/2)$ $C_n(z)$ as in equation (2.1); $K = 1-\exp(-NTU_1/3)$
2.4		Same as case of equation (2.3) with Fluid 2 split into four streams.	$P_1 = 2A-\delta$; $A = (a_0+b_0)C_0(4R_1K)$ $\delta = (a_0^2+b_0^2)C_0(8R_1K)+2a_0b_0\exp(-4R_1K)$ $a_0 = K[1-(K/2)\exp(-4R_1K)]$; $b_0 = K(1-K/2)$ $C_0(z)$ as in equation (2.1); $K = 1-\exp(-NTU_1/4)$
2.5		Two-pass cross-parallel-flow exchanger; Fluid 1 unmixed and twisted; Fluid 2 mixed between passes. Within each pass. Fluid 2 is split into two streams individually mixed.	$P_1 = 2A-R_1A^2-\delta$; $A = (a_0+b_0)C_0(2R_1K)$ $\delta = (a_0^2+b_0^2)\exp(-2R_1K)+2a_0b_0C_0(4R_1K)$ $a_0 = K[1-(K/2)\exp(-2R_1K)]$; $b_0 = K(1-K/2)$ $C_0(z)$ as in equation (2.1); $K = 1-\exp(-NTU_1/4)$
2.6		Two-pass cross-parallel-flow exchanger; Fluid 1 unmixed and twisted; Fluid 2 mixed between passes. Within each pass, Fluid 2 is split into three streams individually mixed.	$P_1 = 2A-R_1A^2-\delta$; $A = (a_0+b_0)C_0(3R_1K)+a_1C_1(3R_1K)$ $\delta = (a_0^2+a_0a_1+b_0^2+a_1^2/6)\exp(-3R_1K)$ $+2a_0b_0C_0(6R_1K)+2a_1b_0C_1(6R_1K)$ $C_n(z)$ as in equation (2.1); $K = 1-\exp(-NTU_1/6)$ a_0, b_0 and a_1 are given by the same expressions as in equation (2.3), but with K as given here.
2.7		Same as case of equation (2.5) but overall cross-counterflow.	$P_1 = (2A-R_1A^2-\delta)/(1-R_1\delta)$ A and δ as in corresponding cross-parallelflow case, equation (2.5).
2.8		Same as case of equation (2.6) but overall cross-counterflow.	$P_1 = (2A-R_1A^2-\delta)/(1-R_1\delta)$ A and δ as in corresponding cross-parallelflow case, equation (2.6).
2.9		Four-pass cross-parallel-flow exchanger, Fluid 1 unmixed and twisted, Fluid 2 mixed.	$P_1 = 2A-R_1A^2-\delta$ $A = (1-R_1K^2\alpha-\alpha^2)/R_1$; $\delta = \alpha[a_0(a_0+a_1)+a_1^2/6]$ $a_0 = K(1-K)+K\alpha$; $a_1 = K^3R_1$ $K = 1-\exp(-NTU_1/4)$; $\alpha = \exp(-KR_1)$
2.10		Four-pass cross-counterflow exchanger, Fluid 1 unmixed and twisted, Fluid 2 mixed.	$P_1 = [1-1/(1+2A+A^2-R_1\delta)]/R_1$ $A = \alpha^2-R_1K^2\alpha-1$; $\delta = \alpha[a_0(a_0-a_1)+a_1^2/6]$ $a_0 = K(1-K)+K\alpha$; $a_1 = K^3R_1$ $K = 1-\exp(-NTU_1/4)$; $\alpha = \exp(KR_1)$
2.11		1–1 TEMA H shell-and-tube exchanger, tube fluid split into two streams individually mixed, shell fluid mixed.	$P_1 = E[1+(1-BR_1/2)(1-AR_1/2+ABR_1)]-AB(1-BR_1/2)$ $A = \{1-\exp[-NTU_1(1+R_1/2)/2]\}/(1+R_1/2)$ $B = (1-D)/(1-R_1D/2)$ $D = \exp[-NTU_1(1-R_1/2)/2]$ $E = (A+B-ABR_1/2)/2$
2.12		Overall parallelflow 1–2 TEMA H shell-and-tube exchanger, shell and tube fluids mixed in each pass at a cross section.	$P_1 = [1-(B+4G/R_1)/(1-D)^4]/R_1$ $B = (1+H)(1+E)^2$; $G = (1-D)^2(D^2+E^2)+D^2(1+E)^2$ $H = [\exp(-2\beta)-1]/(1+4/R_1)$ $E = [\exp(-\beta)-1]/(1+4/R_1)$ $D = [1-\exp(-\alpha)]/(1-4/R_1)$ $\alpha = NTU_1(4-R_1)/8$; $\beta = NTU_1(4+R_1)/8$

Results using Pignotti chain rule

In the preceding section, the out-of-tube fluid was mixed between tube rows or passes. If that fluid is also unmixed between tube rows or passes, the effectiveness cannot be evaluated by the Domingos rules since they cannot handle unmixed streams between tube rows or passes. While the cases of planar unmixed streams between tubes or passes have been analyzed by a number of investigators as mentioned above for overall counterflow, most cases with overall parallelflow have not been investigated. One is analyzed here using the chain rule and is included as equation (2.1). No results have been reported for the cases having twisted unmixed streams between tubes or passes. Seven cases are analyzed here using the chain rules and are reported as equations (2.3)–(2.9) in Table 5.

As summarized in the text, the effectiveness of 1–1 TEMA *H* shell is obtained using the chain rules and is summarized as equation (2.11).

Results using rules for exchangers with one fluid mixed

New results for three exchangers are obtained using these rules. The effectiveness of a four-pass cross-parallelflow exchanger with Fluid 1 unmixed and planar and Fluid 2 mixed is summarized as equation (2.2) in Table 5 and is obtained knowing the effectiveness of a similar exchanger but in overall counterflow as given by Nicole [28]. Similarly, from equation (2.9) for the twisted four-pass cross-parallelflow exchanger, equation (2.10) in Table 5 is derived for the corresponding cross-counterflow configuration. Also knowing the effectiveness of a 1–2 TEMA *H* exchanger in overall counterflow direction [29], the effectiveness of the 1–2 TEMA *H* exchanger in overall parallelflow direction has been obtained here and summarized as equation (2.12) in Table 5.

CONCLUSIONS

In order to determine the effectiveness–*NTU* relationship for highly complex exchanger flow arrangements, some powerful methods have been developed over the last 20 years. Among them are the Domingos rules, Pignotti chain rule and rules for exchangers with one fluid mixed. These methods are briefly discussed in the text. Detailed examples are provided to illustrate the methods using the Domingos rules and Pignotti chain rule. Using these methods, effectiveness–*NTU* explicit formulas are obtained for a total of 18 new exchanger complex flow arrangements and are summarized in Table 5. Because of the space limitations, graphical results are not presented here.

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RELATIONS EFFICACITE–NOMBRE D'UNITES DE TRANSFERT POUR DES ARRANGEMENTS D'ECOULEMENT COMPLEXES DANS LES ECHANGEURS

Résumé—La performance d'un échangeur de chaleur (ou les températures terminales), la taille et les débits de fluide sont reliés sous forme adimensionnelle en fonction de l'efficacité de l'échangeur, du nombre d'unités de transfert et du rapport des débits calorifiques. De telles relations sont essentielles pour le dessin et l'analyse d'un échangeur quelconque. Dans les cent dernières années, beaucoup d'arrangements d'écoulement ont été analysés et rapportés dans la littérature technique. Néanmoins depuis 1969, plusieurs méthodes puissantes ont été développées pour analyser des arrangements compliqués relatifs aux récupérateurs à deux fluides : ce sont le formalisme matriciel, la règle de chaine, et les règles pour des échangeurs avec mélange sur un fluide, parmi d'autres. Ces méthodes sont brièvement résumées avec des illustrations. En les utilisant, 18 nouveaux arrangements d'écoulement dans les récupérateurs sont analysés et les résultats sont présentés en des formules qui supposent constants le coefficient de transfert et les propriétés des fluides. Les résultats résumés ici ainsi que ceux publiés ailleurs fournissent une idée sur les points à étudier dans l'avenir.

WIRKUNGSGRAD/NTU-BEZIEHUNGEN FÜR WÄRMEAUSTAUSCHER MIT KOMPLIZIERTER STROMFÜHRUNG

Zusammenfassung—Das Leistungsvermögen (in Gestalt von Temperaturänderungen) eines Wärmeaustauschers, seine Größe und die Fluidmassenströme werden in dimensionsloser Form verknüpft, wobei sich der Wärmeaustauscher-Wirkungsgrad, *NTU* und das Verhältnis der Wärmekapazitätsströme ergeben. Derartige Beziehungen sind für Konstruktion und Analyse sämtlicher Wärmeaustauscher wesentlich. In den vergangenen 100 Jahren wurden viele Arten der Stromführung in Wärmeaustauschern analysiert; das Ergebnis wurde in der Literatur dargestellt. Seit dem Jahre 1969 wurden jedoch einige sehr leistungsfähige Verfahren für die Analyse komplizierter Stromführungsarten in Zweifluid-Rekuperatoren entwickelt. Dazu gehören u.a. Matrixformalismen, die Kettenregel sowie Regeln für Austauscher mit einseitiger Vermischung. Diese Verfahren werden in der vorliegenden Arbeit kurz zusammengefaßt und anhand von Beispielen erklärt. Unter Verwendung dieser Verfahren werden 18 neue Stromführungsarten in Rekuperatoren analysiert. Das Ergebnis wird in geschlossener Form dargestellt, wobei jeweils der Wärmedurchgangskoeffizient und die Stoffeigenschaften als konstant angenommen werden. Diese Ergebnisse sollten zusammen mit denjenigen aus der zugänglichen Literatur dem Leser einen Hinweis geben, auf welchem Gebiet zukünftige Forschungsaktivitäten erforderlich sind.

СООТНОШЕНИЯ ДЛЯ ЭФФЕКТИВНОСТИ ТЕПЛООБМЕННИКОВ СО СЛОЖНОЙ ОРГАНИЗАЦИЕЙ ТЕЧЕНИЙ

Аннотация—В безразмерном виде представлены рабочие характеристики (или температуры на входе и выходе), содержащие эффективность теплообменника. Такие соотношения являются важными при конструировании и анализе любых типов теплообменников. За последние 100 лет в литературе проанализированы и описаны многие виды организации течений в теплообменниках. После 1969 года было разработано несколько эффективных методов анализа сложных организаций течений в рекуператорах с двумя жидкостями. Среди прочих, данные методы включали матриный формализм, цепное правило и правила для теплообменников с одной смешивающейся жидкостью. Эти методы вкратце обобщены и проиллюстрированы в настоящей работе. На их основе проанализированы 18 новых видов организации течений в рекуператоре, и полученные результаты представлены в виде замкнутых выражений в предположении постоянства суммарного коэффициента теплопереноса и свойств жидкости. Вместе с имеющимся в литературе, полученные результаты послужат ориентиром в будущих исследованиях по данной теме.